IV-8 ON WAVE PROPAGATION IN PERIODIC MEDIA CONTAINING FERRITE

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Periodic structures are used as filters and as interaction structures in microwave tubes. In this paper some of the effects of anisotropic loading are discussed. It is shown that plane structures can be used for separating modes and that proper loading of waveguide circuits with anisotropic media causes their ω - β plots to be shifted. A ring circuit is discussed which has the nonreciprocal property that the phase shift from one port to another is the negative of the phase shift in the reverse direction. In all cases the medium is assumed to be lossless ferrite; however, the conclusions also apply to plasmas and optically active media.

In isotropic periodic media vertically and horizontally polarized modes are degenerate. The presence of ferrite and a longitudinal d-c magnetic field break up this degeneracy.[1] In addition, if there is a d-c magnetic field oblique or transverse to the direction of power propagation, longitudinal components of a-c magnetic field will exist. At the boundary of an anisotropic media cross coupling can exist between the various modes. In a media consisting of slabs with infinite transverse dimensions this behavior can be treated by a 4 x 4 transmission matrix formulation. The elements of this matrix have been derived. For longitudinal or transverse magnetic field the 4 x 4 matrix reduces to two 2 x 2 matrices diagonally within the 4 x 4 matrix, the other elements being zero. Thus for these cases the modes may be treated separately. The results lead to polarization filters which can separate right circularly from left circularly polarized waves for a longitudinal d-c magnetic field and parallel from perpendicularly polarized wave for a transverse d-c magnetic field. Figure 1 shows ω vs. β for plane waves in three media; the first consisting entirely of ferrite, the second of alternate sections each .0025 meters long of air and ferrite, and the third of all air. The ferrite had an f_m of 4.76 GHz and a longitudinal magnetic field strength of 2000 oersteds. The second case demonstrates the filtering property for left and right circularly polarized modes since the passbands for the two modes do not occur at the same frequencies. A uniform ferrite medium also has this property, but only near resonance where the losses are high.

It can be shown that the change in propagation constant of a periodic waveguide is given by[2]

$$\beta - \beta_0 \approx \frac{\iint_{\delta A} [\mu_0 (H_0^* \cdot M + H_0 \cdot M^*)]\, da}{8 S_0}$$

for small perturbations, where S_0 is the power flow in the guide and H_0 the magnetic field before it was perturbed, M is the magnetization of the ferrite and the increment of area is taken over the ferrite. This expression gives opposite signs for the magnitude of the change in β for TE_{01} waves propagating in the forward and reverse directions in rectangular guide, as it does for uniformly loaded guide, when the magnetization is found using the permeability tensor for ferrite. Therefore a periodic structure lacking transverse reflection symmetry can have an $\omega - \beta$ plot which is not symmetrical about the $\beta = 0$ axis. This is never true for an isotropic guide with longitudinal reflection symmetry.

A periodic structure is usually shorted at a plane of reflection symmetry to measure its properties. This works because the forward and reverse propagating modes have the same transverse field function, a property which in general is not true of a ferrite loaded waveguide. Therefore, a ring circuit which was periodic in the θ direction was used to test the conclusion reached from perturbation theory. A ring circuit, unlike a linear circuit with infinite length has discrete resonances. If the number of sections is allowed to go to infinity while the section length is held constant, the number of resonances will go to infinity and the resonances will become points on an $\omega - \beta$ curve. Thus the discrete resonances of the ring circuit can be considered to be points on the $\omega - \beta$ plot of a linear structure, provided the original curvature of the sections was small.

The resonances of the ring circuit must occur for phase shifts totaling multiples of 2π radians around the ring. For an $\omega - \beta$ plot which is symmetrical about the $\beta = 0$ axis each resonance at a positive β is degenerate with a resonance with a negative β of the same magnitude. Shifting the $\omega - \beta$ diagram breaks this degeneracy. With this degeneracy broken, an isotropic radiator in the ring excited at one of the resonances will excite a travelling wave unlike the isotropic case where a standing wave is excited. Therefore, for an anisotropic ring circuit, resonances between sections can be excited with an isotropic radiator whose phase shift can be measured to be multiples of $2\pi/n$ apart, where n is the number of section, while with an isotropic ring circuit only phase shifts of 0 or π can be measured. Also, the magnitude of the field function for an isotropic ring circuit with isotropic excitation will have nulls, but those for an anisotropic ring circuit may not.

Figure 2 shows the ring circuit. The two arms at the top of the picture allow phase shift measurements to be made between two adjacent sections and the bottom arm is the input. The ring on the left forms the top of the circuit and the two rings on the right hold it in place. Figure 3 shows the internal configuration of the assembled circuit. The ferrite had a saturation magnetization of 1700 gauss. The resonances of the ring circuit with and without ferrite loading without d-c magnetic field are shown in Figure 4. Table I, which shows the phase shift per section measured with a bridge circuit demonstrates the shifting of the $\omega - \beta$ plot, and Figure 5 shows corresponding $\omega - \beta$ plot. The phase shift is small on a per section basis but is proportionally larger for a structure consisting of many sections. Figure 6 shows some patterns of the internal electric field squared vs. position around the ring made by the frequency perturbation method.[4] These patterns also show that travelling rather than standing waves exist inside the pass-band which also confirms the shifting of the $\omega - \beta$ plot.

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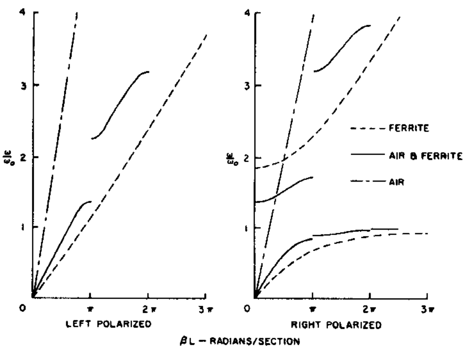
1. D.A. Watkins, Topics in Electromagnetic Theory (New York, John Wiley, 1958, pp. 94-105).
2. B. Lax and K. Button, Microwave Ferrite and Ferrimagnetics (New York, McGraw-Hill, 1962, pp. 589-90).
3. P.R. McIsaac and C.C. Wang, "Interaction Impedance Measurements by Propagation Constant Perturbation," Proc. I.R.E., 48 (May, 1960) p. 908.
4. E.L. Ginzton, Microwave Measurements, (New York, McGraw-Hill, 1957, p. 439).

TABLE I
Theoretical Versus Measured Phase Shift

Theoretical Phase Shift (deg)	Measured Phase Shift (deg)				
	Sample Size (in.)				
	.096 x .091 x .500			.0855 x .084 x .500	.0965 x .144 x .500
	1st Pass Band B_0 = 2400 gauss	2nd Pass Band B_0 = 1200 gauss	1st Pass Band B_0 = 1000 gauss (inner wall)	1st Pass Band B_0 = 2000 gauss	2nd Pass Band B_0 = 1600 gauss
0	0	0	2	6	4
45	42	55	52	45	52
90	101	94	87	95	92
135	141	148	140	114	138
180	176	184	184	183	189
225	226	224	228	230	235
270	262	266	274	272	266
315	314	315	323	298	314



ω VERSUS β FOR THE CIRCULARLY POLARIZED PLANE-WAVE MODE IN A PERIODIC ARRAY OF FERRITE SLABS

FIG. 1

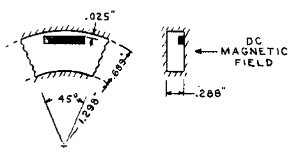


FIG. 3 - Circuit Configuration

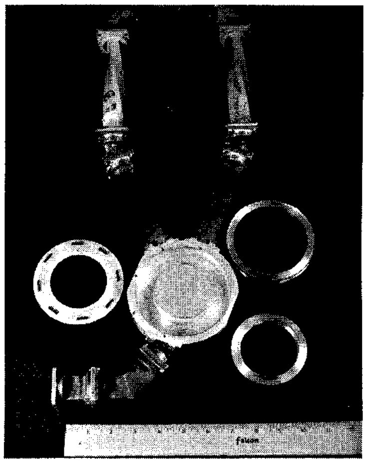


FIG. 2 - Ring Circuit for Phase-Shift Measurements

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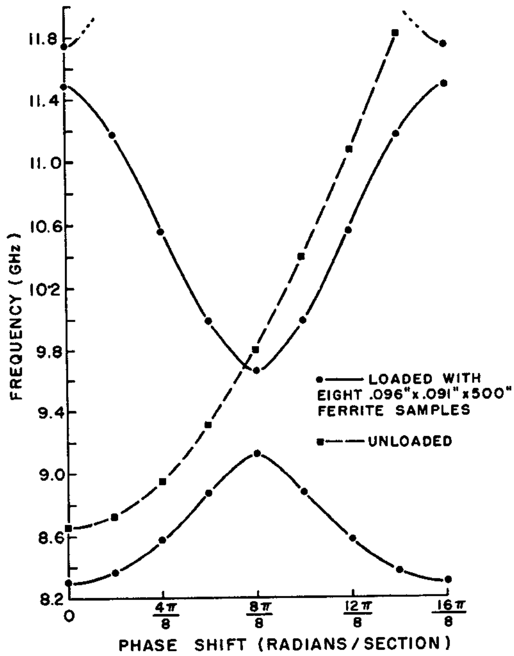

FIG. 4 - Frequency versus Phase Shift for an 8-Section Periodic Ring Circuit Without D-C Magnetic Field

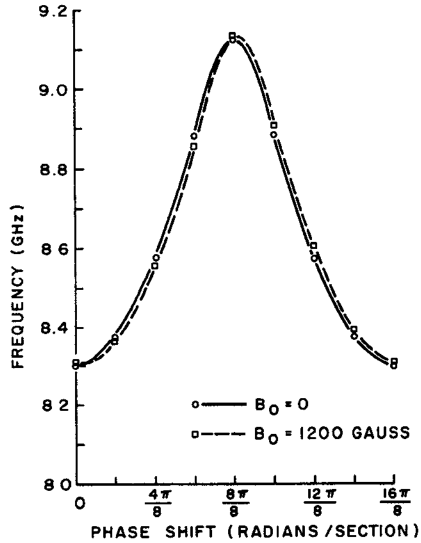

FIG. 5 - Frequency versus Phase Shift for the first Pa Band of an 8-Section Periodic Ring Circuit Loaded wi 0.096" x 0.091" x 0.055" Ferrite Samples

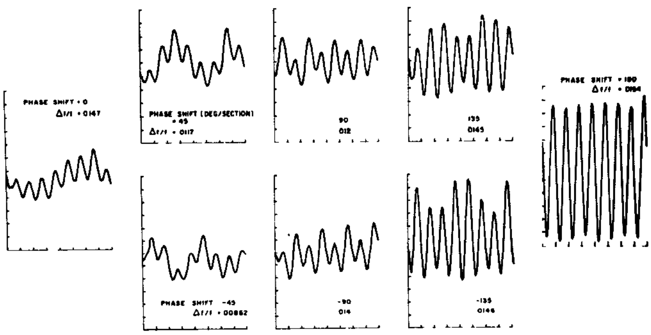

FIG. 6 - E^2 versus Θ for the First Pass Band of an 8-Section Periodic Ring Circuit Loaded with 0.0966" x 0.096" x 0.500" Ferrite Samples, with B_0 = 1200 Gauss